

The Neurobehavioral Basis of Parallel Individuation and Numerical Approximation: An EEG Study

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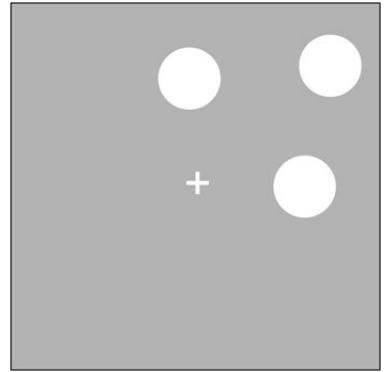
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A Graduate School of Education, Health & Psychology

Presentation, Cognitive Science Society 2022
July 29, 2022

Two Distinct Number Systems^{1, 2, 3}

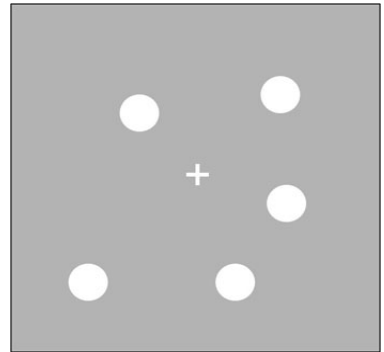
Small # system: 1, 2, 3¹

- Subitizing, Parallel individuation ²
- Object Tracking System (OTS)



Large # system: 4...¹

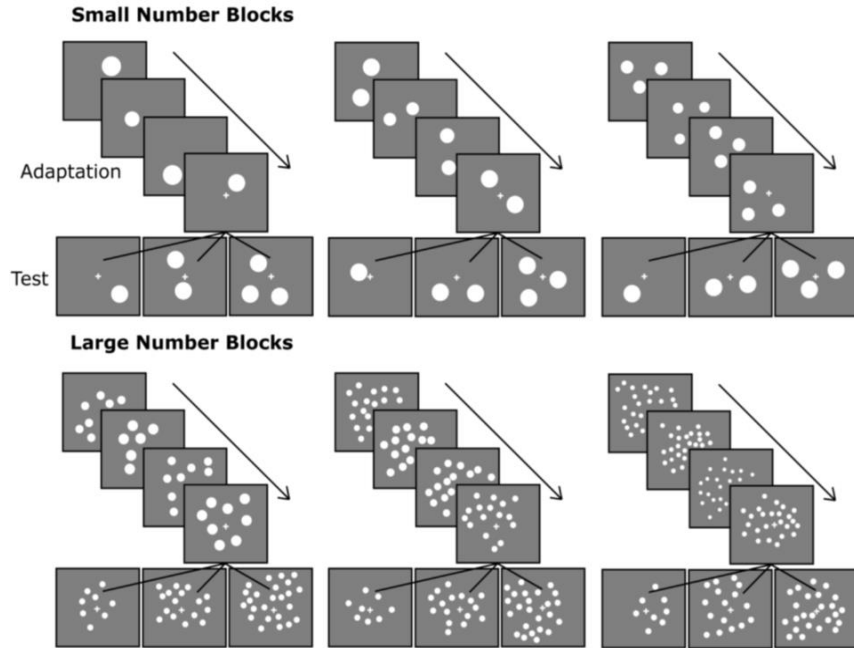
- Weberian numerical magnitude estimation
- Approximate Number System (ANS)



Previous Research in Numerical Cognition

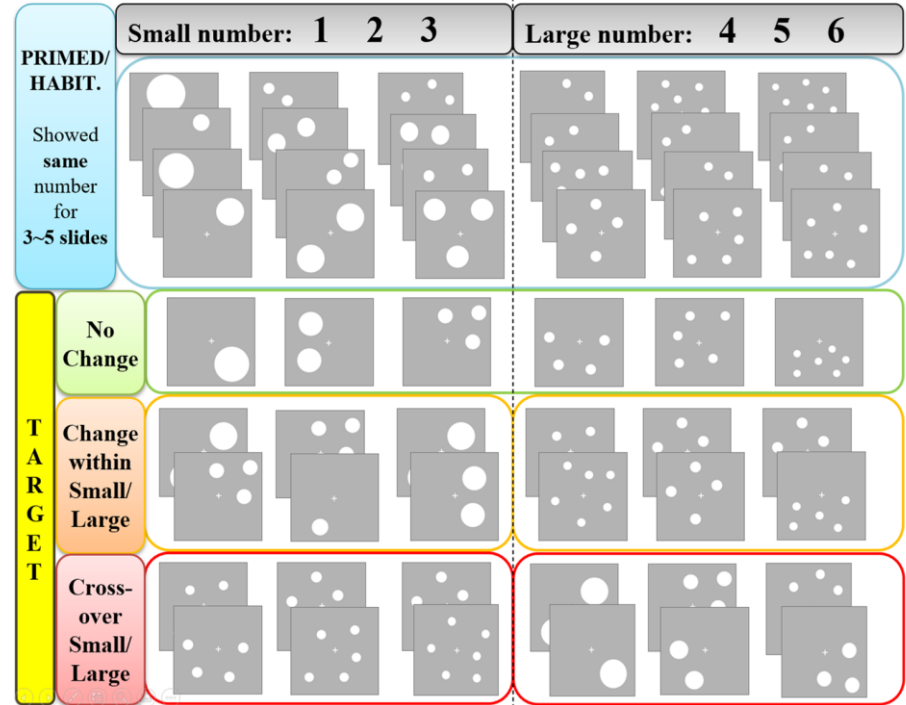
- **Behavioral** oddball paradigms: Change detection studies have traditionally relied on participants, performance recorded as response time (RT) & accuracy.
- **EEG** studies: Brain's electrocortical activity recorded during change detection tasks & analyzed as Event-Related-Potentials (ERP).
- Current issues: Much of numerical change detection research uses **large** numbers (10~100). ^{3, 5-6}
- Also, most ERP studies focused on the **magnitude (size)** of change, but few have studied the **direction** of change. ^{5, 6}

Hyde & Spelke (2012) ³



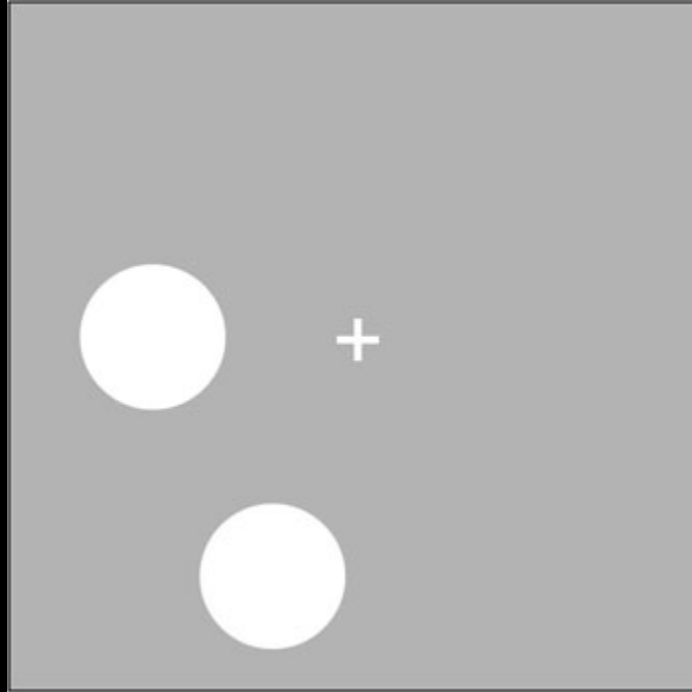
- **Discontinuous** #: Small (1, 2, 3) // Large (8, 12, 16)
- **Passive** observation of change during EEG.

Current Study: Tang et al. (2022) ⁸

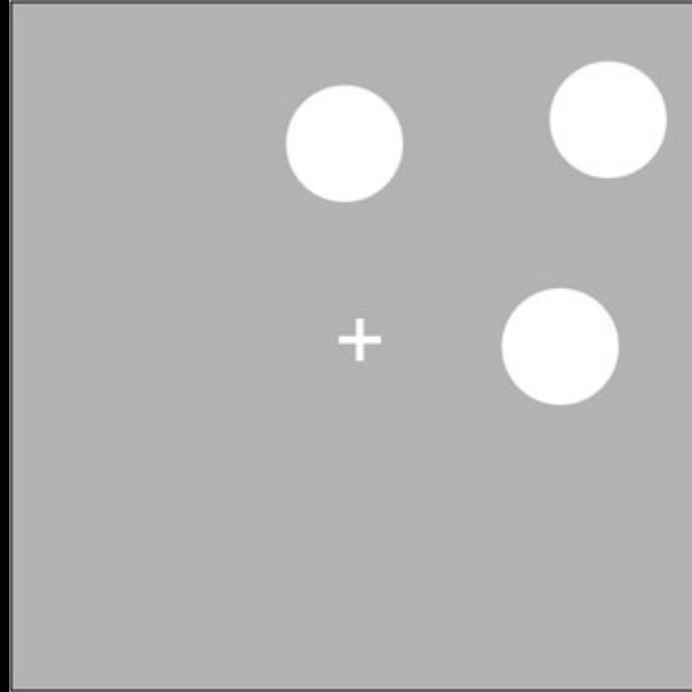


- **Continuous** #: Small (1, 2, 3) ↔ Large (4, 5, 6)
- **Active** detection of numerical change (key press)
- See next slides for animations of what participants saw.

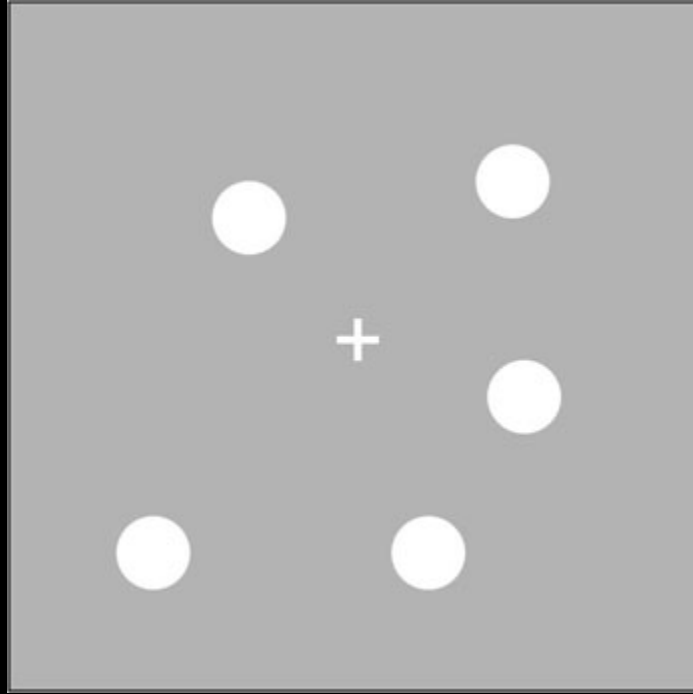
2-3: Increase Small-Small



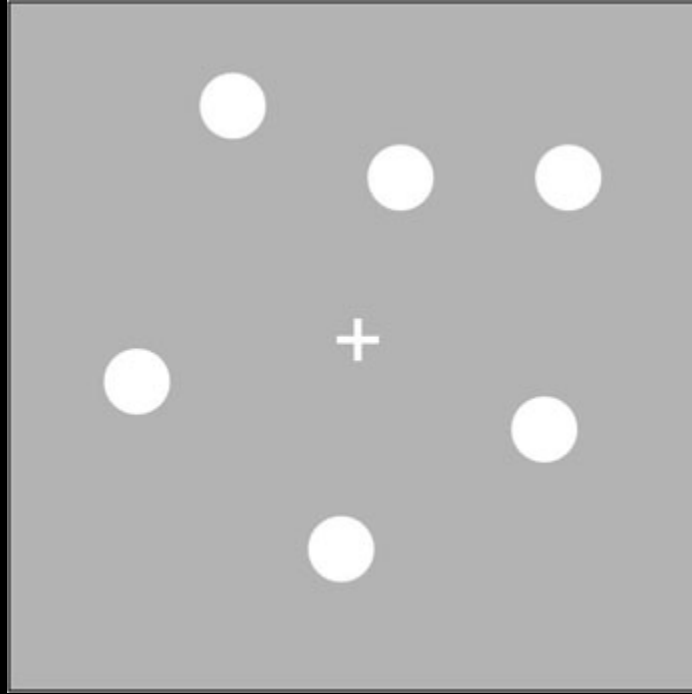
3-2: Decrease Small-Small



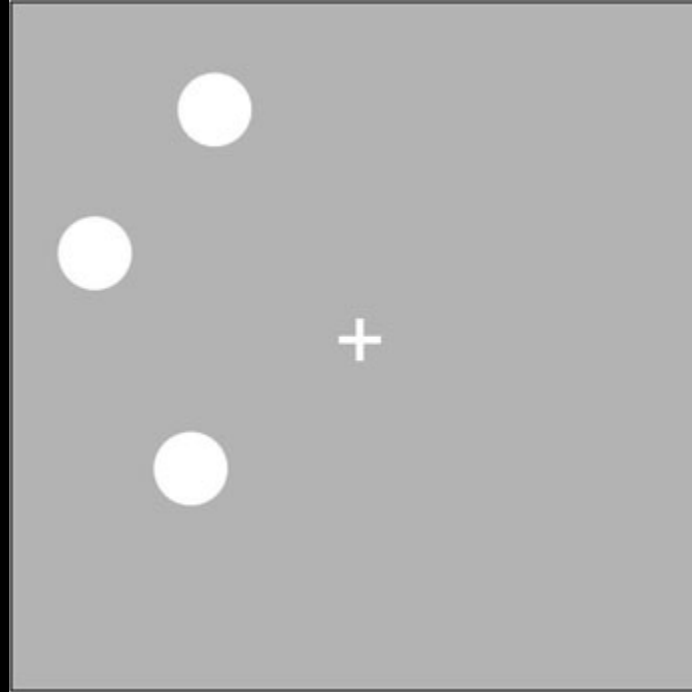
5-6: Increase Large-Large



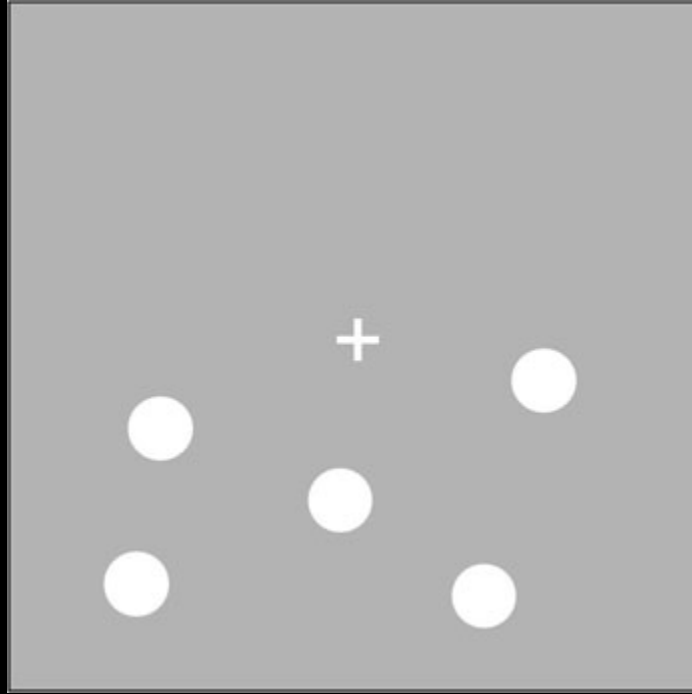
6-4: Decrease Large-Large



3-5: Increase Small-Large

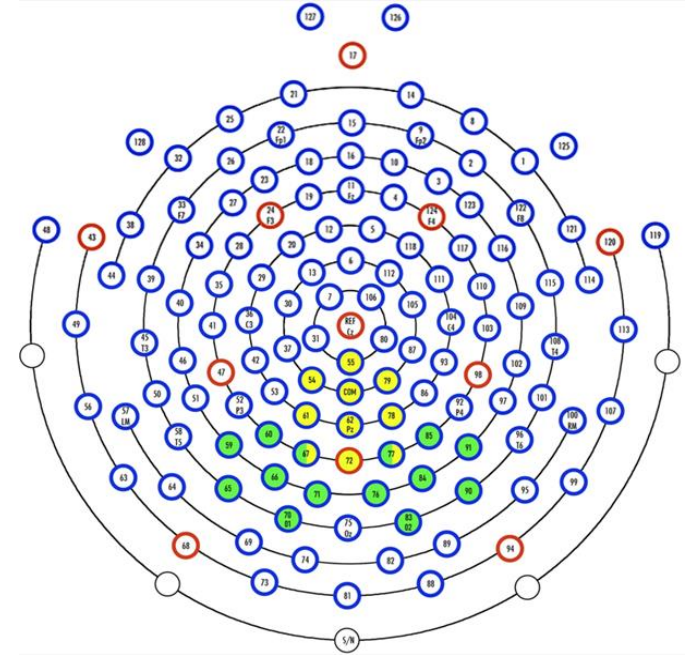


5-3: Decrease Large-Small



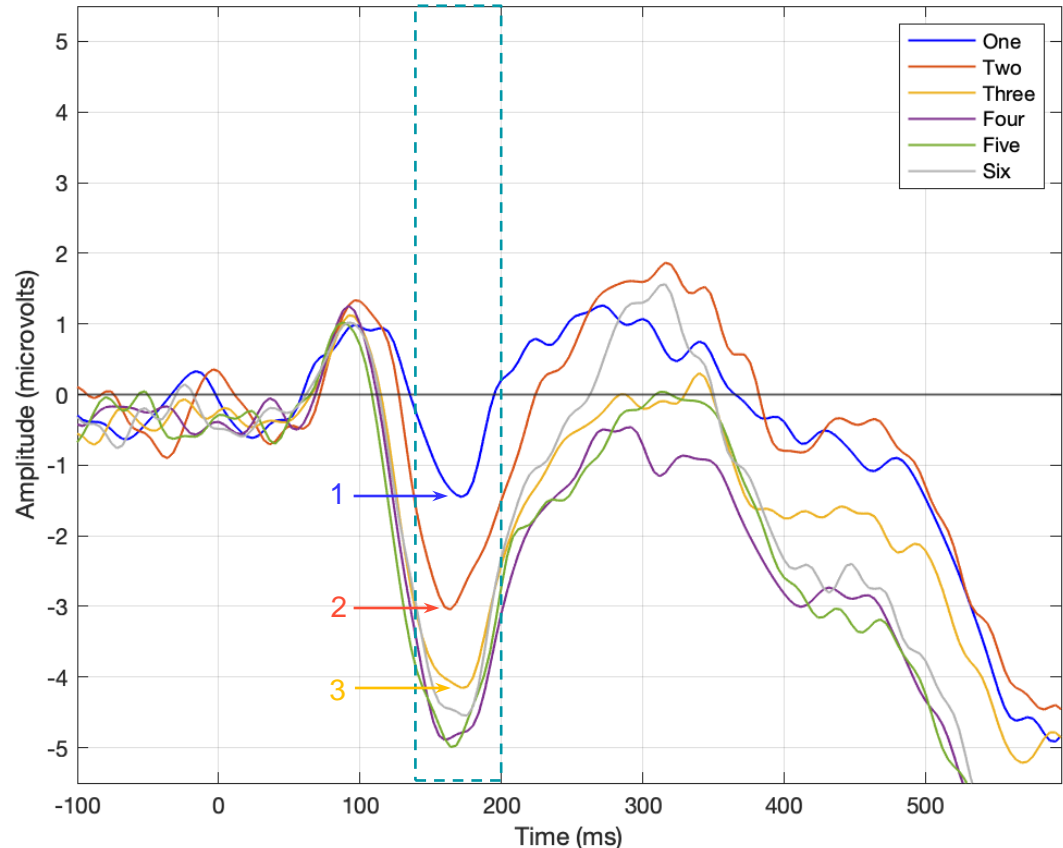
Procedure (contact for more info)

- 15 RH participants
- 128-channel EGI GSN system ERPs recorded over:
 - **POT** (Parietal Occipital Temporal): Green
 - **Pz** area: Yellow
- Recorded variables:
 - N1 (POT area) to cardinal values **1~6**
 - Behavioral change detection: RT & Accuracy
 - N1 (POT area) and P3b (Pz area) to change detection.
- Predictor variables:
 - **Direction:** Increasing (e.g., 1→2; 5→6) or Decreasing (e.g., 5→4; 3→2)
 - **Size:** Small-Small; Large-Large; Crossover (Small-to-Large; Large-to-Small)



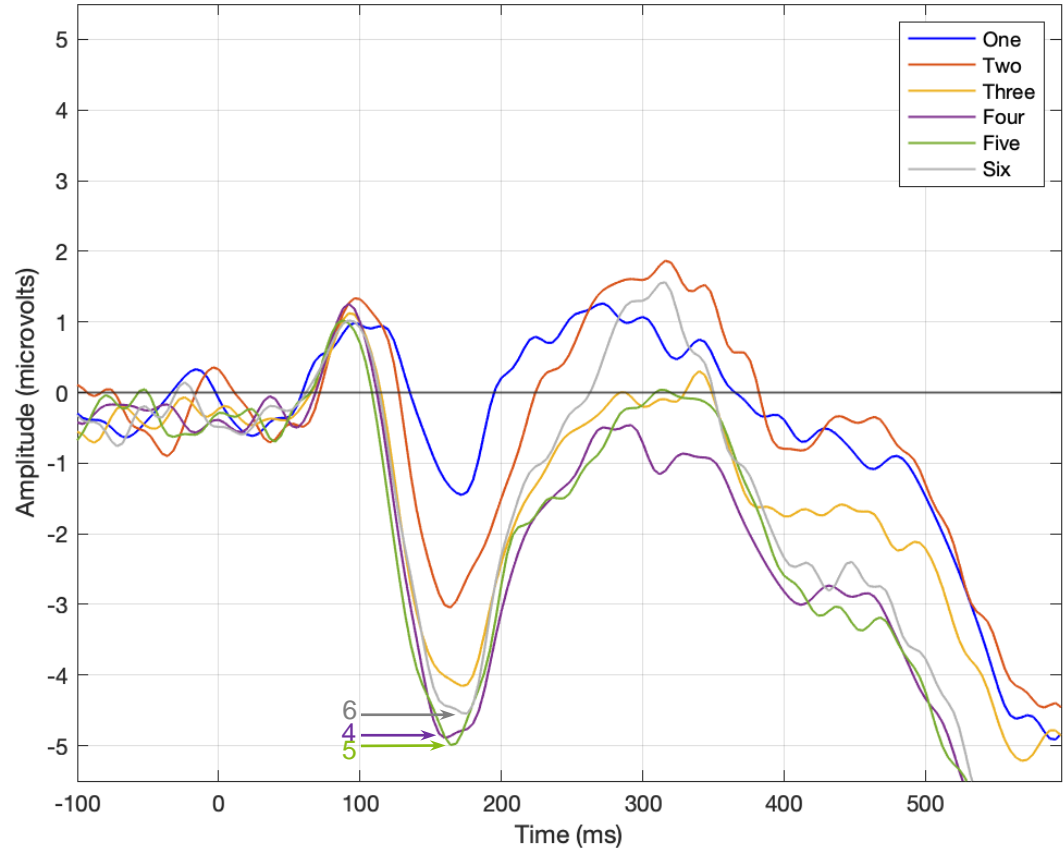
Amplitude of N1 (125 - 200ms) over POT: Cardinal Value

- In trials without numerical change = No behavioral response.
- EEG recordings to different cardinal values (1 ~ 6): Measured from the N1 ERP over the POT area.
- Found distinct ERPs in small numbers (subitizing range).
- N1 amplitude is scaled to values “1”, “2”, “3”.



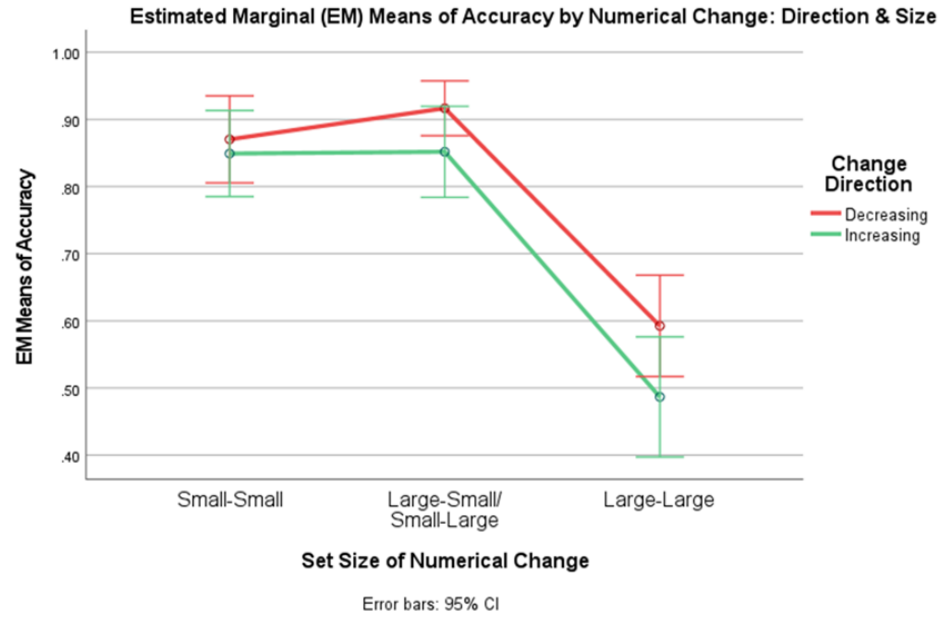
N1 (125 - 200ms): Cardinal Value

- Yet, N1 ERPs are not distinct for larger numbers (4, 5, 6).
- This justifies our categorization of 1~3 as “**Small**” and 4~6 as “**Large**”.
- *Proposed:* As more objects are loaded into early working memory, N1 amplitudes become stronger.
- Important: Diff. cardinal values did not have significantly different N1 latencies = No indication of a serial process.



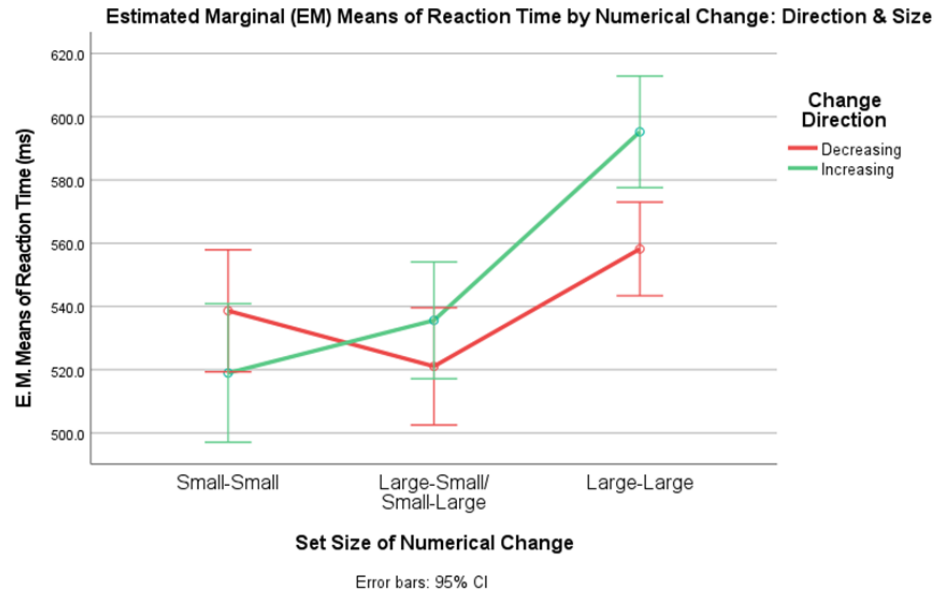
Accuracy by Numerical Change: Size & Direction

- **Size:** Accuracy decreases with larger numbers, as numerical change detection gets harder ($p < 0.000$).
- **Direction:** Accuracy is higher for **Decreasing** change compared to **Increasing** change. ($p < 0.001$).
- This contrasts previous studies that found better accuracy for Increasing (but they used numbers 10~70).^{5, 6}



Reaction Time

- **Size*Direction:** Sig. int. effect ($p < 0.000$), where RT was longest for Increasing-Large number change ($p < 0.05$), followed by Decreasing-Large.
- **Size:** RT increases with larger numbers, as numerical change detection gets harder ($p < 0.000$).
- **Direction:** There were trends that **Decreasing** conditions have shorter RTs, except in the Small-Small condition ($p < 0.01$).
- This contrasts previous studies that found shorter RTs for Increasing (but they used numbers 10~70).^{5, 6}



Context Updating Theory of P3b ⁷

- Related to updating one's working memory in change detection paradigms.
- Incoming sensory input → Evaluated as being the **same** or **different** from the previous context ⁷.
- If **different** → Elicits an updating of the given neural representation at P300.

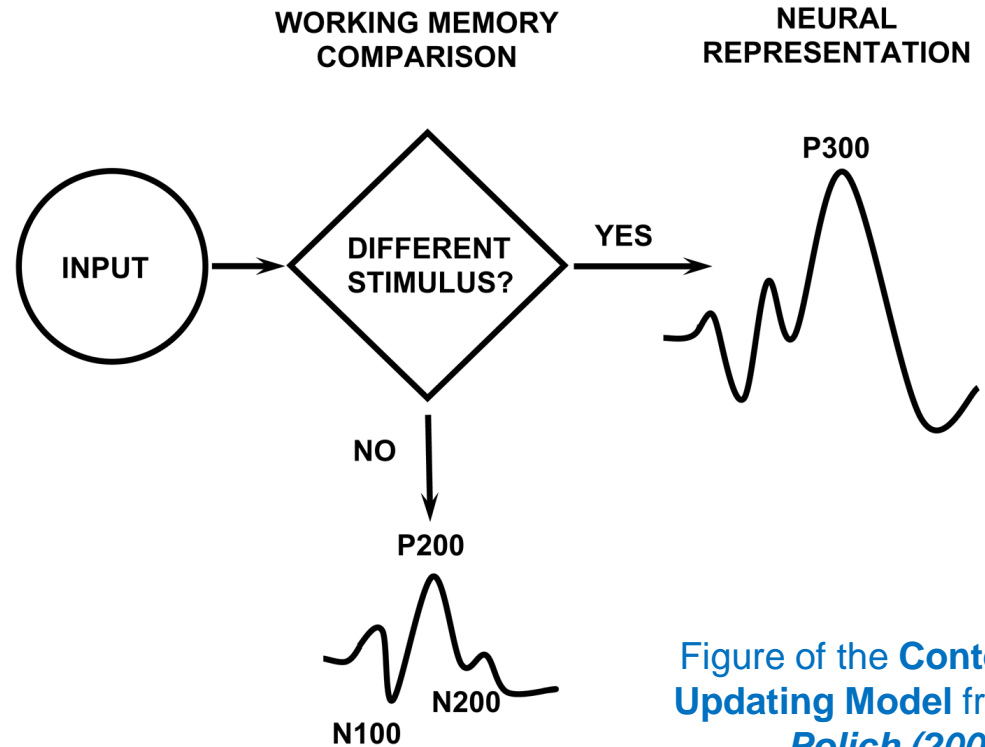
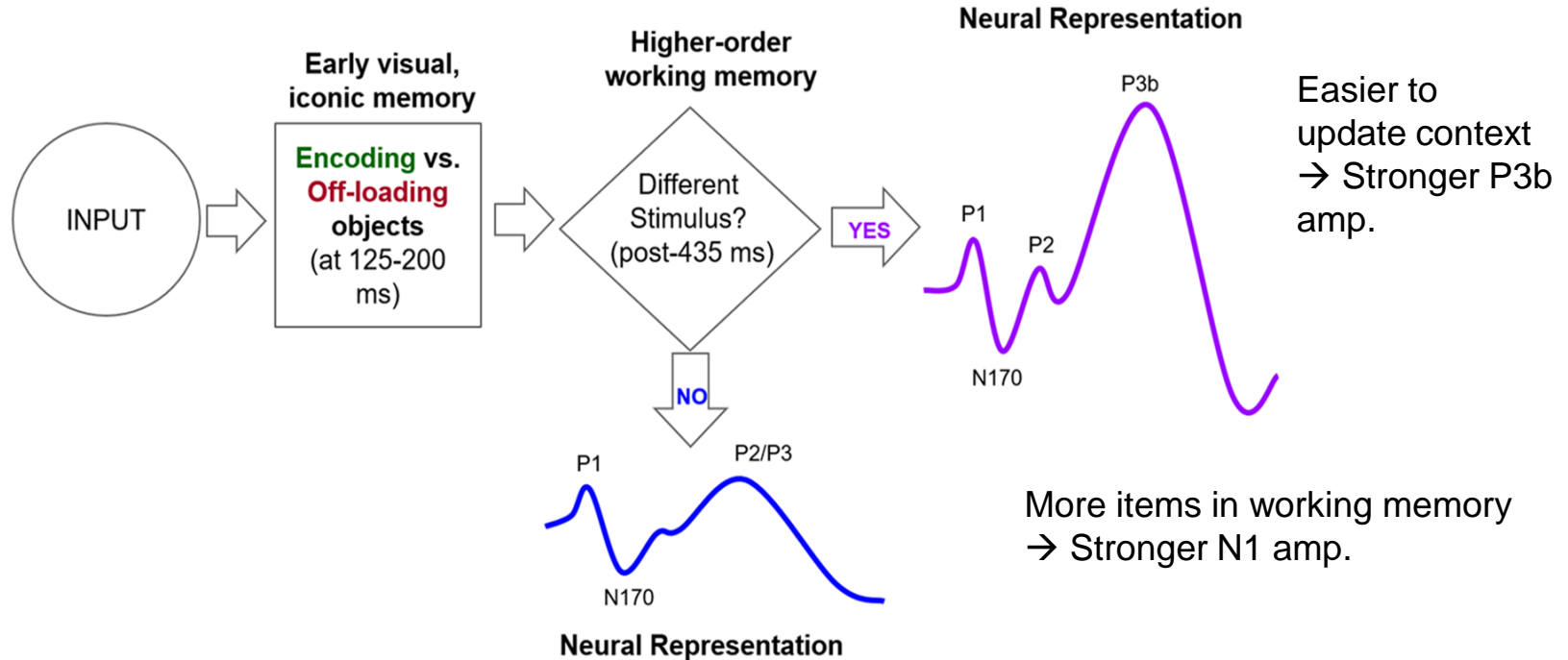


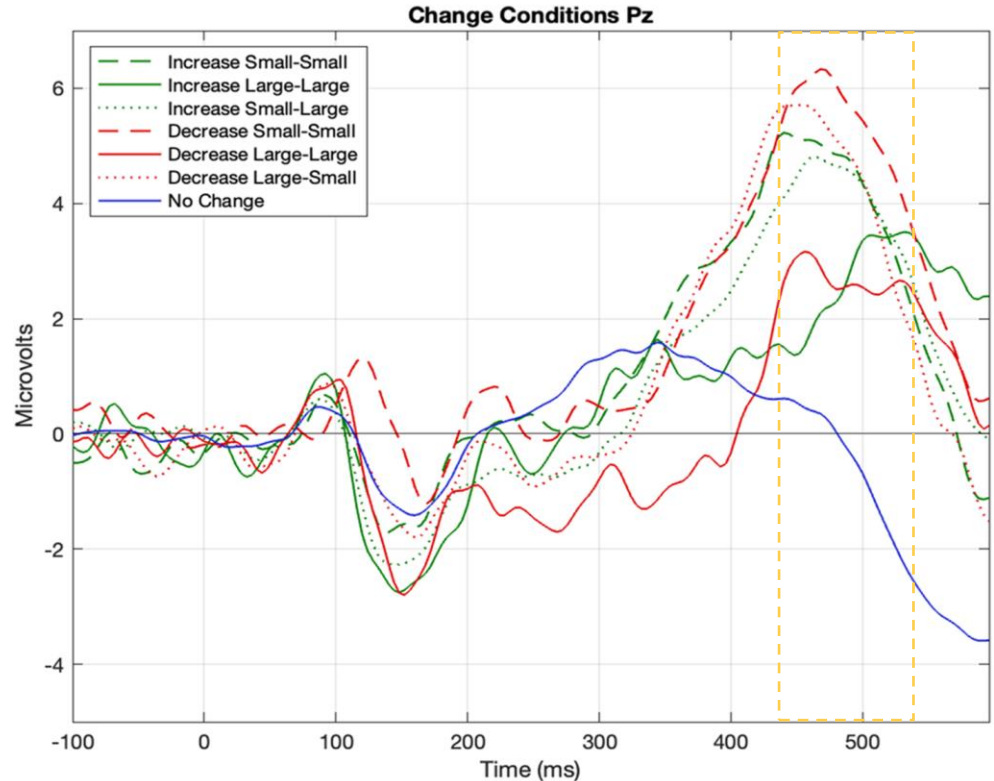
Figure of the **Context Updating Model** from *Polich (2003)*⁷

Proposed Model of Early & Later/Higher-order Working Memory in Context-Updating



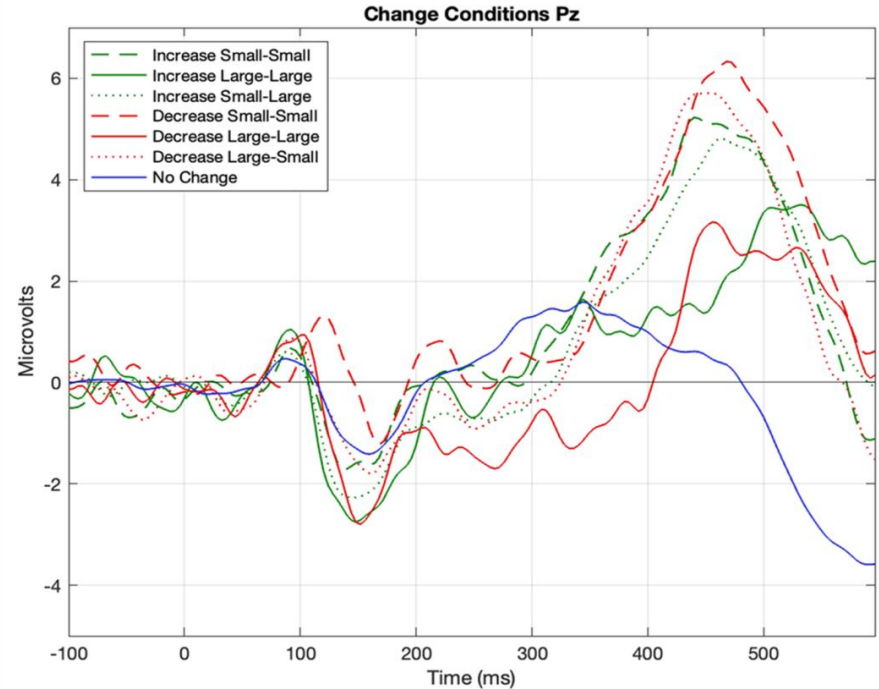
Amplitude of P3b (435-535ms) over mid-Parietal (Pz) area: Later cognitive ERP of Change Detection

- **No Change:** Weak P3
- **Decrease Small-Small:** Strongest P3b amp.
- Increase & Decrease Large-Large: Weaker P3b amp.
- Only “Size” mattered for P3b amp.
- Polich⁷: Easier to update the context → Higher P3b Amp.

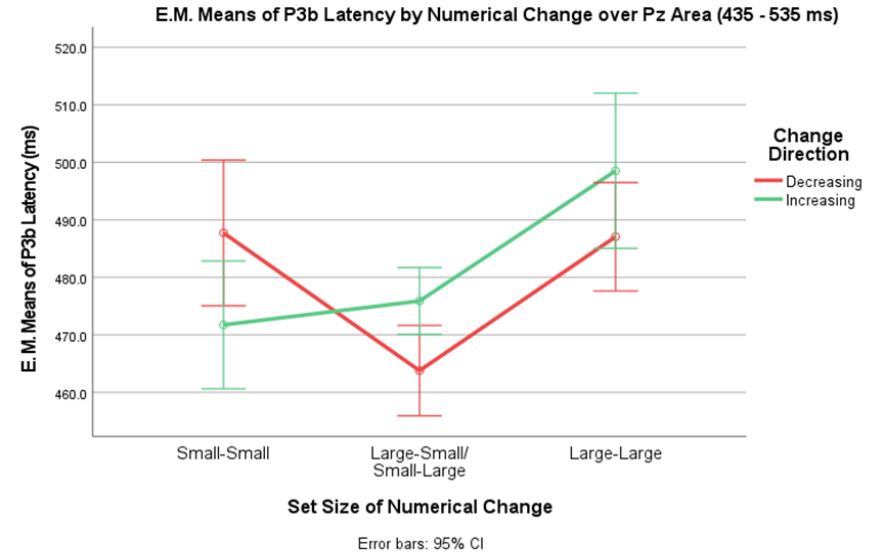
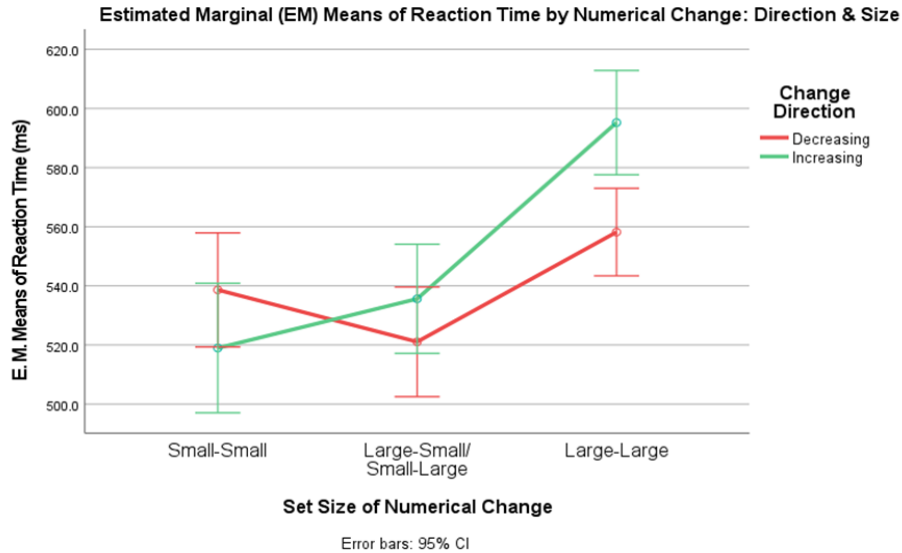


P3b Latency over Pz: Later cognitive ERP of Numerical Change Detection

- Latency was highest in Large-Large conditions, followed by the Small-Small conditions, and least for the crossovers ($p < 0.0001$).
- **Direction*Size**: Significant ($p < 0.001$); in the small condition, **Decreasing** trials had higher latency than **Increasing** trials.



Linking Brain & Behavior: RT and P3b Latency



Similar results for reaction time & peak latency of the P3b ERP!

Conclusions

- Our findings mirror previous research^{1, 4}: Scaling of N1 ERP amplitudes to small numbers (1~3), but not large numbers – even when both categories are continuous on the number line (1~6).
- Previous studies^{5, 6} on change direction found consistent superiority of **Increasing** changes in set size for larger numbers over a wide range (10~100).
- Our study uses a narrower range (1~6) and found better performance for **Decreasing** set sizes that interact with set sizes.
- ERP distinctions reflect a categorical break in Direction and Size, reflecting working memory loads (N1) and ease of context-updating (P3b).
- Aligns with Polich's context-updating model⁷, where working memory representations differ between small and large numbers, as well as increasing and decreasing numerical change.
- Suggests a neural basis for the differentiation of small vs. large number perception at early stages of processing, and a later stage that involves more complex numerical processing that is employed in our numerical change detection task.

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1. Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences*, 8(7), 307–314.
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8. **Tang-Lonardo, J.**, Bisbee, N., Jain, M., Kirby, E., Kim, S. B., Abdelrahim, S., Gerami, S., Sezcon-Cepeda, D., Coffel, M., & Gordon, P. (May 26 - 29, 2022). *The Neural Mechanisms of Parallel Individuation and Numerical Approximation*. [Conference Poster] Association for Psychological Science (APS) 2022 Annual Convention, Chicago, IL, USA.

Thank you!

For questions, please

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The Neural Mechanisms of Parallel Individuation and Numerical Approximation

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Theoretical Overview

Research on numerical cognition using behavioral, neuroimaging, developmental and cross-cultural methods, converges on the conclusion that there are two distinct systems for the perception of numerical quantity.

1. A "small number" system (1-3) involving subitizing and object-to-object comparison (Gordon, 1984).
 2. A "large number" system (4+) that is based on Weberian accuracy magnitude estimation/numerical approximation.
- Previous EEG research in numerical cognition:
- Previous studies have found ERP N1 negativities associated with numerical values in the posterior Parieto-Occipital-Temporal (POT) region (Temple & Posner, 1968).
 - Hyde & Spelke (2012) employed a passive numerical viewing task to examine ERPs associated with changes within the small number (1-3) and large number (8-16-24) range. Participants viewed 4 dot displays with the same number of dots (adaptations), followed by a test array with either the same number or a small vs. large change. No changes crossed between small and large number categories.
 - Hyde & Spelke (2012) examined test times that involved changes (or no change) following the adaptation phase. Within the small number range (1-3), the N1 ERP showed scaling of magnitude with numerical quantity.
 - A later positivity/discrimination ratios: P3p for small numbers (8-16-24); P3 for large numbers (1-3).
 - Rationale for electrode analysis choices was based upon the tendency to observe larger P3b amplitudes in the parietal areas during change processes (Hyde & Spelke, 2012; Poon, 2011).

Study Design

Adapted Design from Hyde & Spelke (2012):

- Our study (Fig. 1) involves a sequential presentation of dot arrays in "Small" (1, 2, 3) and "Large" (4, 5, 6) numerosities to participants.
- Stimulus duration: 250 ms
- Interstimulus Interval: 750 - 1250 ms
- In each trial, to prime/habituate the brain to a numerical value, the same value was presented for three to five slides.
- Followed by a Target slide from either of two conditions:
 - a) No Change (Same number)
 - b) Change Within Small or Large number (e.g., 2 to 5, 5 to 6)
 - c) Cross-over between Small and Large numbers (e.g., 3 to 4, 2 to 6, 6 to 3)

Fig. 1: Current Study Design & Examples of Numerical Stimuli

Fixing Boredom Problems in Design:

- Participant boredom was a serious concern when replicating the passive viewing procedure from Hyde & Spelke (2012).
- To actively engage participants, our study required participants to detect changes in the numerical value, and to press a key when such changes occurred. Number of trials between testing blocks and 3 and 5 so as to be unpredictable.
- Participants were updated on their number of correct responses between testing blocks. At the end of the experiment, each participant was given a lottery scratch-off card for every 6 items they got correct.
- Data from both change trials and no change trials were used in the ERP analysis.

Looking for Categorical Small-Large Differences within a Narrower Range (1 to 6):

Target #	1	2	3	4	5	6
1	1	2	3	4	5	6
2	1	2	3	4	5	6
3	1	2	3	4	5	6
4	1	2	3	4	5	6
5	1	2	3	4	5	6
6	1	2	3	4	5	6

Pair of quantities tested: the Prime-habituated number appears first, followed by the Target number.

Fig. 2: Pairs of quantities tested: the Prime-habituated number appears first, followed by the Target number.

Procedure

- 15 right-handed adults (4 males), aged 23-43 years (mean = 27.7) participated. Told to press a key whenever they detected a numerical change in the dots.
- EEG data was examined to extract Event Related Potentials (ERPs) associated with Habituation to small (1, 2, 3) and large (4, 5, 6) cardinal values.
- Reaction times, accuracy and ERPs were examined.
- Numerical Change conditions: Within Small, Within Large, Crossover Small-to-Large, Crossover Large-to-Small
- Change Direction variable: Increasing (e.g., 1-2, 5-6) or Decreasing (e.g., 5-4, 3-2)
- Numerical Change Distance: Difference of 1, 2, 3 (see Fig. 2 for all numerical pairs)

EEG Acquisition and Processing

- Apparatus: 128-channel EEG Geodesic Sensor Net with High Impedance Amplifier
- EEG Recording
- Recorded in a shielded sound attenuating chamber
- Amplified analog voltages were stored digitally
 - The signals were recorded at 1 - 100 Hz bandpass filtered & sampled and digitized at 250 Hz using Net-Station EEG acquisition software and an AD amplifier
 - Impedance of electrodes was kept below 50 kΩ
 - Individual voltages were referenced to the average across all electrodes.
- EEG Data Processing
 - 40 Hz low-pass digital filter applied
 - Segmentation of 800ms length epochs, starting 100 ms before onset of stimuli
 - Artifact rejection
 - Epochs associated with the same category (i.e., no change, within small, within large, or crossover) for Detrending and increasing change conditions were averaged within subjects
 - EEG recordings were re-referenced to average and the baseline correction was performed to 100ms interval preceding the stimulus onset.
 - Responses were averaged within participants.
- Time windows and ERP components:
 - Time Slice 1 (N1T0): 125-200 ms
 - Time Slice 2 (P3b): 435-535 ms
- Montages for ERP components (see Fig. 3)
 - Pz area: 78, 77, 72, 61, 54, 55, 79
 - Parieto-Occipital-Temporal (POT) junction: 66, 55, 69, 60, 67, 71, 70 (Left); 64, 75, 77, 85, 91, 90, 83 (Right)



Fig. 3: Map of electrode groupings used for averaging and analysis: Pz area (yellow) and POT (green)

ERPs to Cardinal Numerical Values

- N1T0 (125-200 ms) was measured over the Parieto-Occipital-Temporal (POT) junction (see Fig. 3), where there was a separation of ERPs within the subitizing numerical range (1-3), but not beyond (4-6).
- As cardinal value increases, more objects are encoded in early visual working memory, leading to stronger N1T0 ERP amplitudes.
- Post-hoc comparisons and repeated pairwise contrasts revealed that the N1T0 amplitudes for One is different from Two, Two is different than Three, and Three is different than the later cardinal values (all $p < 0.05$).
- In the large number range (Four and above), the amplitudes were not discernible from each other.
- Stronger amplitudes were observed over the right hemisphere (consistent with Hyde & Spelke, 2012), but there are smaller N1T0 patterns on the left.
- Relative magnitudes of N1T0 deflections corresponded to ordered numerical magnitudes within the small-number range, but not within the large number range.
- This scaling of the N1 ERP to numerical magnitude replicates Hyde & Spelke (2012).
- Scaling is clearer in our data, and the categorical break between "1" and "2" followed by "3" and "4-6" is apparent.
- Fig. 4 is based on ERP data from only "same number" adaptation trials, not from numerical change trials.

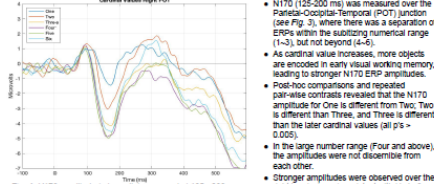
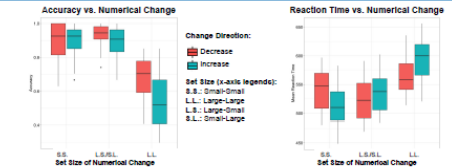


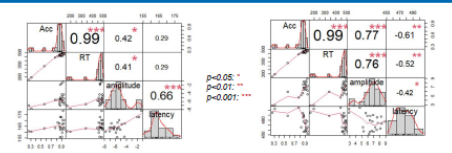
Fig. 4: N1T0 amplitude (microvolts) measured at 125-200ms for cardinal magnitudes (1-6) in prime-habituated trials without change in numerical value over the right POT region.

Behavioral Results



- The effects of Size ($p < 0.001$) and Direction ($p < 0.01$) on Reaction Time were significant. Size and Direction had a significant interaction effect ($p < 0.001$).
- Reaction time was longest for Increasing-Large number change ($p < 0.05$), followed by Decreasing-Large.
- Accuracy is lower for increasing change compared to decreasing change.
- Accuracy was lowest for Large-Large conditions.
- This is reflected in their later P3b deflection (see Fig. 10).

Linking Behavior to ERP Data



- Accuracy and RT are very strongly positively correlated with each other.
- Accuracy and RT are moderately correlated with N1T0 amplitude, and N1T0 amplitude and latency have a moderate to high positive correlation.
- Accuracy and RT are strongly correlated with Amplitude and Latency of P3b ERP signal.
- Accuracy and RT are negatively correlated with latency and positively correlated with amplitude, and P3b amplitude and latency have a moderate negative correlation.

Context-Updating & Working Memory Model

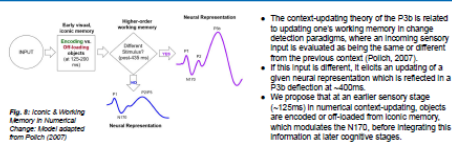
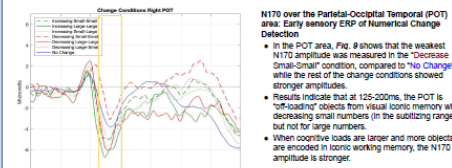


Fig. 8: Iconic & Working Memory Model diagram showing the flow from Early sensory encoding to Higher-order working memory and Neural Representation.

Numerical Change: ERP Results



- N1T0 over the Parieto-Occipital Temporal (POT) area: Early sensory encoding of the numerical change detection.
- In the POT area, Fig. 9 shows that the weakest N1T0 amplitude was measured in the "Decrease Small-Small" condition, compared to "No Change", while the rest of the change conditions showed stronger amplitudes.
- Results indicate that at 125-200ms, the POT is "off-loaded" objects from visual iconic memory with decreasing small numbers (in the subitizing range), but not for large numbers.
- When cognitive loads are larger and more objects are encoded in iconic working memory, the N1T0 amplitude is stronger.
- P3b over the mid-Parietal (Pz) area: Later cognitive ERP of Numerical Change Detection condition.
- Fig. 10 shows that over the Pz area at 435-535 ms, "Decrease Small-Small" has the strongest P3b amplitude, while Increases and Decrease Large-Large have weaker P3b amplitudes.
- As the number stays the same in No Change, there is a weak P2 signal instead.
- Change within small numbers (1-3) had the highest P3b amplitude, while change within large numbers (4-6) had the lowest amplitude ($p < 0.01$), with no differences based on change direction.
- P3b latency was the highest in Large-Large conditions, followed by the Small-Small conditions, and least for the crossovers ($p < 0.001$).
- While there were no direct effects for direction of change, we found significant interaction effects ($p < 0.001$). Within the small condition, decreasing trials had higher latency than increasing trials.
- When change detection is easier, it is associated with higher P3b amplitudes, reflecting context-updating processes.

Fig. 10: P3b amplitudes for different numerical change conditions over the mid-parietal area with Small and Crossover Sets showing stronger P3b amplitudes; Large Sets showing weaker and later P3b peaks, and No Change conditions showed the earliest weakest P3b.

Conclusions

- Our findings suggest a neural basis for the differentiation of small vs. large number penetration at early stages of processing, and a later stage that involves more complex numerical processing that is employed in our numerical change detection task.
- In contrast to Hyde & Spelke (2012), who examined distinct small (1, 2, 3) vs. large (8, 16, 24) numbers, we examined a smaller numerical range (1-6), so that small (1-3) vs. large (4-6) contrasts were along a numerical continuum.
- Within this continuous range, we found N1T0 amplitudes commensurate with cardinal values in the small range (1, 2, 3), but not in the large range (4, 5, 6), where the process of encoding/off-loading objects in memory determines the amplitude strength, suggesting that numbers in the subitizing range are individuated in working memory.
- Distortions in P3b waveforms also reflect a clear categorical break between increasing vs. decreasing, and small vs. large numbers, where essential number change conditions have stronger amplitudes than number, large number conditions, suggesting more difficulty with updating the context in the latter.
- Overall findings align with the context-updating model (Poon, 2007; see Fig. 8), where working memory representations differ between small and large numbers, as well as increasing and decreasing change.

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Current Study: Specific Number Pairs

Target # \ Primed #	1	2	3	4	5	6	
1	Same #	inc.SS	inc.SS	inc.SL	N/A	N/A	
2	dec.SS	Same #	inc.SS	inc.SL	inc.SL	N/A	
3	dec.SS	dec.SS	Same #	inc.SL	inc.SL	inc.SL	
4	dec.LS	dec.LS	dec.LS	Same #	inc.LL	inc.LL	
5	N/A	dec.LS	dec.LS	dec.LL	Same #	inc.LL	
6	N/A	N/A	dec.LS	dec.LL	dec.LL	Same #	
Pairs of numerical change (e.g., "1→2" = the Primed # is "1", followed by the Target # as "2")	No change (Same number)	Small → Small (SS)		Large → Large (LL)		Cross-over Small-to-Large (SL) & Large-to-Small (LS)	
		inc.	dec.	inc.	dec.	increase	decrease
	1→1; 2→2; 3→3; 4→4; 5→5; 6→6	1→2; 1→3; 2→3	2→1; 3→1, 3→2	4→5; 4→6; 5→6	5→4; 6→4; 6→5	1→4; 2→4; 2→5; 3→4; 3→5; 3→6;	4→1; 4→2; 4→3; 5→2; 5→3; 6→3

Overall Results:

Variables	Direction	Size	Direction * Size
RT	$p = 0.006$ (0.03)	$p < 0.000$ (0.37)	$p < 0.000$ (0.13)
Accuracy	$p = 0.013$ (0.03)	$p < 0.000$ (0.35)	$p < 0.000$ (0.12)
POT N1 Amp	$p < 0.000$ (0.12)	$p < 0.000$ (0.17)	$p < 0.000$ (0.10)
POT N1 Lat	$p = 0.003$ (0.04)	$p = 0.602$ (0.0)	$p = 0.076$ (0.02)
Pz P3 Amp	$p = 0.218$ (0.0)	$p = 0.002$ (0.06)	$p = 0.234$ (0.01)
Pz P3 Lat	$p = 0.59$ (0.0)	$p < 0.000$ (0.21)	$p = 0.014$ (0.12)

- p-values for significance
- Effect sizes in parentheses